43 p.

NASw-729

UNPUBLISHED PRELIMINARY DATA

N64-17792

CODE-1

CR-53319

LTV RESEARCH CENTER

OTS PRICE

XEROX $ 4.60 ph

MICROFILM $ 1.49 mf.

ENCLOSURE (

LET

0-71000/4L-18

1409762

Ling-Temco-Vought, Inc., Dallas, Tex.

N64 Conf

ON THE TURBULENT SHEAR FLOW
OF AN ELASTICO-VISCOUS FLUID *

~~Report No. 0-71000/3R-28~~ per. auth.

December 1963 43 p refs 24

(NASA Contract NASw-729)
(NASA CR-53319; Rept.-0-71000/3R-28)
OTS: $ 4.60 ph, $ 1.49 mf

Prepared by: C. S. Wells
C. S. Wells
Research Scientist

Approved by: John Harkness
J. Harkness
Senior Scientist

H. B. Gibbons
H. B. Gibbons
Associate Director

* Presented at the AIAA ~~Included in the program of the American Institute of Aeronautics and Astronautics~~ Aerospace Sciences Meeting, New York, 20-22 January 1964 ~~New York, New York.~~

17792

ABSTRACT

A

General categories of possible fluid systems are described, including Newtonian, purely viscous non-Newtonian, and elastico-viscous non-Newtonian. The recent interest in the turbulent shear flow of real elastico-viscous fluids and their relation to the aerospace sciences and technologies are discussed. An analysis of this problem is presented in terms of the viscous fluid properties, the Prandtl mixing length concept, and a stability criterion. Experiments are described which provide data on frictional drag and velocity profiles in fully developed pipe flow of several fluids presumed to be elastico-viscous. The experimental results are compared to the analysis with respect to the thickness of the viscous sublayer, shape of the velocity profile, variation of the mixing length constant and the effects of pipe diameter.

AUTHOR

TABLE OF CONTENTS

INTRODUCTION

An elastico-viscous fluid is one which exhibits both elastic and viscous characteristics. In order to describe the properties of such fluids, it is necessary to specify the relation of the basic variables for the deformation of materials as functions of time, i.e., how the shear stress (as a function of time) is related to the strain (as a function of time). The properties can be determined by choosing a "mechanical model" which phenomenologically duplicates the observed time dependence of the material. Such a model, for example, is a spring in series with a dashpot.

The above view of elastico-viscous materials (fluids) must now be reconciled with the accepted general classification of fluids into two categories; either Newtonian or non-Newtonian. The following set of terms refers to the types of fluid systems of interest here:

A. Newtonian (only viscous characteristics, with stress proportional to rate of strain, or shear rate).

B. Non-Newtonian

   1. Purely viscous (only viscous characteristics, stress not proportional to shear rate).

   2. Elastico-viscous (both viscous and elastic characteristics, stress may or may not be proportional to shear rate).

The turbulent shear flows to be discussed are those which are constrained, both fully constrained as in the case of flow in a pipe or channel and half constrained as in the case of a boundary layer flow.

The turbulent shear flow of elastico-viscous non-Newtonian fluids is a new problem in fluid mechanics. The complex fluid mechanical situation of turbulent shear flow coupled with the complex fluid properties of elastico-viscous fluids yields experimental results which do not agree with the empirical relations evolved for the turbulent flow of Newtonian fluids. Therefore, new approaches in the analysis of turbulent shear flow are needed; and, since analyses of turbulent flow are still largely the application of simple models or criteria to experimental data, extensive experimental data on the flow of these fluids are needed also. The purpose of this report, then, is twofold: (1) to define better the actual problem of interest through separating the new and interesting facets from those which are already understood; and (2) to provide, through experiments and in the light of a new analysis, more basic information about the momentum transport process through a turbulent shear layer.

In a physical sense the fluids of interest here are liquids composed of small concentrations (1 percent by weight or less) of high molecular weight polymers dissolved or hydrated in Newtonian solvents (although other types of fluids have these characteristics). These fluids appear to be very little different physically from the solvents alone, other than having increased viscosity. The study of the flow of such liquids has direct applications in the aerospace field. The application can be shown through the most easily observable characteristic of the turbulent flow of these elastico-viscous liquids, which is a large reduction in drag due to fluid friction compared with the flow of a Newtonian, or purely viscous non-Newtonian, liquid with the same density and viscosity (or distribution of viscosities with shear rate). This single aspect makes an understanding of the flow necessary in order to use this drag reduction beneficially or simply to make accurate predictions of the performance of such fluids in ordinary engineering applications. An example of the former use of these fluids is the transpiration of elastico-viscous fluids into the turbulent boundary layer on marine vehicles in order to reduce skin friction drag or boundary layer induced noise. Examples of the latter use are, in particular; flow through ducts of liquid rocket fuels which have elastico-viscous properties; and, in general, turbulent flow of these fluids in any system where viscous drag, severity of mixing, pumping capacity, or flow rate are major considerations.

Most of the serious study of turbulent flow of elastico-viscous fluids has begun within the past five years. The major contributions have come in the form of experiments in the simplest and most versatile flow apparatus available - a long circular pipe.

All of the previous investigators recognized their experimental fluids to be non-Newtonian, to the extent that the viscous properties were examined and were found to be non-linear with shear rate. In fact, all of the fluids described in the literature were found to give a decreasing viscosity (defined as the shear stress divided by the shear rate) for increasing shear rate when subjected to a simple shearing motion. This characteristic is called "shear-thinning" (or less descriptively, "pseudoplastic"). This led to attempts at correlation of the turbulent pipe flow data by use of a flow model valid for non-Newtonian fluids which are purely viscous and isotropic, given in terms of two constants which are properties of the fluid,

$$\tau = a \left(\frac{du}{dy} \right)^n , \tag{1}$$

which describes shear thinning fluids for $n < 1$, Newtonian fluids for $n = 1$.

No attempts to include the effects of elasticity were made in the previous investigations because in some cases the authors were not fully aware that the fluids were elastico-viscous and, more significantly, because an analysis of the rheology of these fluids has not been developed sufficiently. It should be noted that sketchy evidence on the fluids used in the turbulent flow experiments indicate that they possessed small elasticity. The lack of experimental data on the elastic properties of these fluids makes it all but impossible to make practical use of the general rheological models which describe such fluids. More will be said below concerning the rheology of these fluids.

Although the interest in this paper is focused on the basic turbulent momentum transport process, the state of knowledge to the present can be traced through the studies of frictional drag of elastico-viscous fluids. In 1948 Toms[1] performed turbulent pipe flow experiments with a dilute polymer solution, which Savins[2] later showed to represent a drag reduction over that predicted by viscous theory. It should be noted here that Dodge and Metzner[3], by analysis and experiments, showed that a purely viscous non-Newtonian fluid described by Eq. (1) would provide a small amount of drag reduction. Dodge and Metzner also conducted the same experiments with a synthetic polymer, carboxymethylcellulose, which was later shown to be elastico-viscous[4] and which showed a much greater drag reduction than predicted or observed for purely viscous non-Newtonian fluids. Shaver and Merrill[5] and Ousterhout and Hall[6] showed the same trends to greater drag reduction by experiments with elastico-viscous fluids. Various observations and conclusions regarding the different trends in frictional drag were made by these investigators, but more detailed data on the flow itself was needed to achieve any greater understanding of the situation. Clearly, more information about the basic flow properties was needed also, but very few results applicable to the problem of turbulent flow of liquids with such small elasticity as used in the experiments have appeared in the literature.

In view of the previous investigations, it was felt that a series of turbulent pipe flow experiments was needed to provide sufficient data to analyze the problem in terms of the known variables of turbulent flow.

ANALYSIS

Because the functional dependence of the elastic properties on the flow parameters cannot be given, as will be discussed below, a complete dimensional analysis of the turbulent shear flow cannot be made. If, instead, it is assumed that the elasticity affects, in a measurable way, the model for the turbulent shear layer which has proved successful for Newtonian fluids, then an analysis can be performed in terms of the measurable quantities. In this way, the assumptions leading to the analysis can be checked experimentally.

The model used for the turbulent shear flow or, more descriptively, the transport of momentum through a turbulent shear layer to a constraining surface, consists of a viscous sublayer (not necessarily thin) near the wall and turbulent flow in the outer part of the layer. The sublayer and turbulent layer are assumed to be affected predominantly by viscous and turbulent shear stresses in each region, respectively, with the extent of either region determined by the stability of the viscous flow.

The analysis of such a model can be performed in terms of measurable flow quantities if the following assumptions are made:

(1) elasticity modifies the momentum transport process in the turbulent region, but the general Prandtl mixing length concept applies,

(2) flow in the viscous sublayer can be described from a knowledge of the viscous properties of the fluid,

(3) the extent of the viscous sublayer is represented by a stability criterion relating the viscous stress to the turbulent stress.

Fluid Properties

When one considers the rheological properties of elastico-viscous fluids, it is necessary to consider a model of some generality, since there are few experimental measurements which allow simplifying assumptions to be made. There is no analogy to Newton's law of viscosity for fluids with elasticity. Several models, including those of Oldroyd[8], Rivlin-Ericksen[9,10] and Coleman-Noll[11] are being used for various types of experiments[12,13,14] to determine elastic properties of fluids. The lack of experimental data on the dependence of elasticity on frequency, scale, and isotropy of turbulent motion, however, prevents the use of these general mathematical models to analyze turbulent motion.

If, as was assumed above, the flow in the sublayer can be described from the viscous properties, and if it is further assumed that the fluid is isotropic, then the power-law expression of Eq. (1) can be used to analyze the flow in the sublayer. This formulation, of course, permits both shear-thinning and Newtonian viscous behavior.

Generalized Velocity Profile

Although elastic properties are not used explicitly, the elasticity is assumed to affect the mixing process, as described by the Prandtl mixing length concept. Thus, the outer velocity profile can be related, through the mixing length theory, to the elasticity.

Acting, then, on the assumption that the same general considerations for turbulent mixing hold for both inelastic and elastic fluids, the shear stress in turbulent flow can be written, following Prandtl[15] in terms of the gradient of the time-averaged velocity:

$$\begin{aligned} \tau &= A \frac{\partial \bar{u}}{\partial y} \\ &= \rho\epsilon \frac{\partial \bar{u}}{\partial y} \end{aligned} \qquad (2)$$

which describes the stresses present in the case of mean parallel flow, where,

$$\bar{u} = \bar{u}\,(y), \quad \bar{v} = \bar{w} = o.$$

This means that τ is an "apparent" shear stress parallel to the streamlines and is composed of both laminar and turbulent shear stresses. The shear stress due to turbulent mixing is assumed to be large with respect to the laminar shear stress. This, then, presents A and ϵ as "virtual" viscosity and kinematic viscosity coefficients, respectively, due to the predominant turbulent stresses present. There follows now a summary of Prandtl's concept of a length which can be related to the virtual viscosity. Some aspects of the concept are physically indefensible; however, it has been proved useful. Eq. (2) is useful in that it demonstrates the use of the Prandtl mixing length concept which will be summarized below.

In order to use the relation for shearing stress given by Eq. (2) it is necessary to find a relation between the virtual viscosity and the mean velocity field. Following Prandtl, it is possible to state through dimensional analysis an expression for the shearing stress in terms of the mixing length, ℓ.

$$\tau_t = \rho \ell^2 \left| \frac{d\bar{u}}{dy} \right| \frac{d\bar{u}}{dy} \quad . \tag{3}$$

The mixing length, ℓ, may be supposed to be something like the mean free path for microscopic motion of molecules except, of course, ℓ is related to macroscopic motion of lumps of fluid. The mixing length was actually conceived from the consideration of a two-dimensional turbulent shear flow. ℓ is proportional to the distance which a lump of fluid having the mean velocity of the layer to which it belongs must move perpendicular to the direction of flow. The criterion for movement in this direction is that the difference between its velocity and the mean velocity of the new layer should equal the mean deviation of velocity in the turbulent flow.

The reasoning used by Prandtl in arriving at the mixing length concept is mentioned above because it is felt that the reasoning itself should hold as well for one type of turbulent mixing as another; therefore, even if there should be a difference in the mixing processes between Newtonian and non-Newtonian fluids, Prandtl's general dimensional analysis should still be valid. The concept has proved to be useful for calculating many types of flows of Newtonian fluids. The general usefulness has been attributed primarily to the similarity of the mixing of the various flows. If Eq. (3) is substituted into Eq. (2) it is found that

$$\epsilon = \ell^2 \left| \frac{d\bar{u}}{dy} \right| \quad . \tag{4}$$

Although the mixing length is an unknown, Eq. (4) is of great help in determining the shear stress because ℓ is not a property of the fluid, but a purely local function. It is assumed that, near the wall, the mixing length is proportional to y, thus

$$\ell = ky, \tag{5}$$

where k is a dimensionless constant which must be determined from experiments.

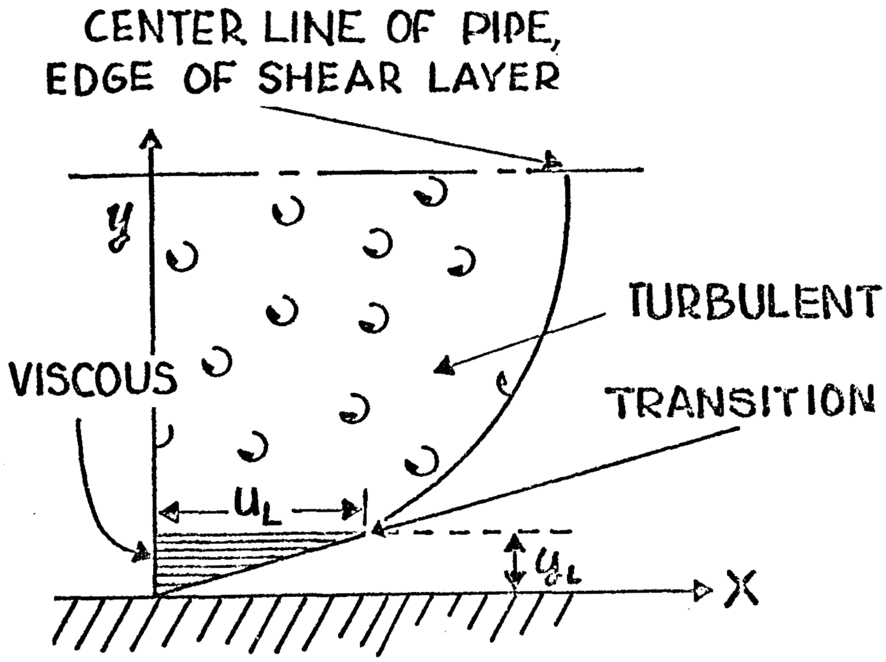


Figure 1 Two-Dimensional Turbulent Shear Layer

Using the second assumption, that the flow in the viscous sublayer can be described from the viscous properties only, a generalized expression can be developed for the sublayer. Referring to Fig. 1, the sublayer extends from the wall ($y = 0$) to $y = y_L$ and is described by a linear velocity profile and a constant shear stress equal to that at the wall, τ_o. Thus, since the sublayer is affected by primarily viscous stresses, Eq. (1) can be written for the velocity, u, at a distance from the wall, y, through the sublayer,

$$\tau_o = a \left(\frac{u}{y} \right)^n . \qquad (6)$$

Defining the friction velocity, $u_*^2 = \tau_o/\rho$, allows the rearrangement of Eq. (6) to give

$$u_*^2 = \frac{a}{\rho} \left(\frac{u}{y} \right)^n ,$$

or,

$$\frac{u}{u_*} = \left(\frac{\rho \, u_*^{2-n} \, y^n}{a} \right)^{\frac{1}{n}} \tag{7}$$

Eq. (7) is valid through the sublayer. The velocity profile in the sublayer can then be described by

$$\phi = \eta_n^{\frac{1}{n}} \qquad (y \leq y_L) \, , \tag{8}$$

where:

$$\phi = \frac{u}{u_*}$$

$$\eta_n = \frac{\rho \, u_*^{2-n} \, y^n}{a} \quad .$$

It should be noted that η_n is a Reynolds number written in terms of the friction velocity, distance from the wall, and the molecular fluid properties. Eq. (8) reduces to the velocity profile in the viscous sublayer for Newtonian fluids ($n = 1$, $a = \mu$).

If, as assumed, the edge of the sublayer is located by the stability of the sublayer, then a criterion proposed by van Driest and Blumer[16] for Newtonian fluids can be extended to include the effects of elasticity. The criterion states that the ratio of turbulent to viscous stress required for transition from laminar to turbulent flow to occur is constant. Writing this expression across the turbulent shear layer in terms of Eqs. (1) and (3) gives

$$\left[\frac{\rho \, \ell^2 \left(\frac{du}{dy} \right)^2}{a \left(\frac{du}{dy} \right)^n} \right]_{y = y_L} = \text{constant} \tag{9}$$

Making the approximation of Eq. (5) gives

$$\left[\frac{\rho k^2 y_L^2 \left(\frac{du}{dy} \right)^{2-n}}{a} \right]_{y = y_L} = \text{constant},$$

which can be rearranged after substitution of τ_o and u_* to yield

$$k \left(\frac{\rho y_L^n u_*^{2-n}}{a} \right)^{\frac{1}{n}} = \text{constant}, \tag{10}$$

or

$$k \left(\eta_{n_L} \right)^{\frac{1}{n}} = \text{constant}. \tag{11}$$

It is interesting to note that Eq. (11) predicts what has been observed experimentally for Newtonian fluids, i.e., the product of k and η_{n_L} is constant for virtually all types of turbulent shear flows; indeed both k and η_{n_L} are constants, being universally 0.4 (0.36 to 0.41) and 11.5, respectively.[15] Anticipating the experimental results for elastico-viscous fluids, it is suggested that the constant on the right-hand side of Eq. (11) is actually the "universal" constant and that if the mixing-length constant, k, is changed, then the sublayer thickness will change accordingly.

Carrying the analysis into the turbulent region of the shear layer, assuming that the viscous sublayer flow breaks up immediately into turbulent flow at y_L, Eq. (3) can be written in terms of the wall shear stress:

$$\tau_o = \rho \ell^2 \left(\frac{du}{dy} \right)^2 , \tag{12}$$

where the time-averaged notation is dropped since no instantaneous values will be considered here. The combination of Eqs. (5) and (12), together with Prandtl's approximation of constant shear stress through the turbulent region, yields

$$\frac{du}{dy} = \frac{u_*}{ky} , \tag{13}$$

which can be integrated to give

$$\frac{u}{u_*} = \frac{1}{k} \ln y + C, \tag{14}$$

where C is a constant of integration.

Eq. (14) can be rewritten in terms of the generalized coordinates as

$$\phi = \frac{2.3}{nk} \log_{10} \eta_n + C_1 \quad (y \geq y_L) \tag{15}$$

The constant on the right side of Eq. (11) is evaluated for Newtonian fluids, being 4.6, and Eqs. (8) and (11) are applied to Eq. (15) as a single boundary condition at $y = y_L$. Eq. (15) can then be written in terms of the generalized velocity and normal variables, and the mixing length constant:

$$\phi = \frac{1}{k}\left[\frac{2.3}{n} \log_{10} \eta_n + 4.6 - 2.3 \log_{10} \frac{4.6}{k}\right] \quad (y \geq y_L) \; . \tag{16}$$

It should be noted that the von Karman mixing length theory[17] gives the same result as Eq. (16) if Eqs. (8), (11) and (13) are anticipated as boundary conditions. It also should be noted that Eq. (16) reduces to the expression for Newtonian fluids given by Prandtl[15] as

$$\phi = 5.75 \log_{10} \eta_n + 5.5 \tag{17}$$

Summarizing, the velocity profiles in the two assumed regions of the turbulent shear layer are given by:

$$\phi = \eta_n^{\frac{1}{n}} \quad (y \leq y_L) ,$$

and

$$\phi = \frac{1}{k}\left[\frac{2.3}{n} \log_{10} \eta_n + 4.6 - 2.3 \log_{10} \frac{4.6}{k} \right] \quad (y \geq y_L)$$

It can be seen that the two profiles intersect at $y = y_L$, but with different slopes. Experiments with turbulent shear flow of Newtonian fluids show a transition region where a gradual change from the sublayer profile to the turbulent region profile takes place; however, the data approach the formulations further into the viscous and turbulent regions.

EXPERIMENTS

Apparatus and Procedure

The polymer used in this study was guar gum, modified to the specifications of the Western Company and designated commercially by that company as J-2P. Guar gum is a natural polymer which hydrates in water. The guar molecule is essentially a straight chain with high molecular weight, and is chemically classified as a polysaccharide. When fully hydrated in water, small concentrations (0 to .5%) of the guar appear only to increase the gross viscosity of the water; although, at higher concentrations (.5 to 5%) the fluid exhibits both increased gross viscosity and gross elasticity. In addition, at the higher concentrations, the fluid will climb a rotating rod, rather than form a vortex around it. This phenomenon indicates a certain type of normal stress present and is predicted by one of the models for elastico-viscous fluids[8]. With these qualitative indications of elasticity for the higher concentrations, it was assumed that the fluids used possessed some elasticity at the lower concentrations.

Four concentrations of the J-2P in water were used in the flow experiments: 0.05, 0.1, 0.2, and 0.4 percent by weight. The variation of shear stress with shear rate as the fluid undergoes simple shear was determined with a Fann V-G coaxial cylinder viscometer in the range of 3 to 1,000 sec^{-1}, and with a modified Merrill-Brookfield coaxial cylinder viscometer in the range of 10,000 to 200,000 sec^{-1}. This range of shear rates extends slightly beyond the largest value of shear rate encountered in the pipe flow experiments.

The pipe flow experiments were carried out in a recirculating system with two sections of smooth extruded aluminum tubing of alternate pipe sizes of 0.65 and 1.43 inches inside diameter . A twenty foot entrance length was provided for both pipes to ensure full development of the flow ahead of the test section. The test section was instrumented with static pressure ports located at several intervals to measure the pressure drop due to friction, and with a total pressure probe mounted on a traverse so that, when coupled with a static pressure port in the same plane, the dynamic pressure in the stream could be measured as a function of radial position. The outside diameter of the probe tip was

0.042 inches. A positive displacement flow meter was used to measure the volume flow rate during the experiments. The two centrifugal pumps placed in series enabled water to be pumped in the larger pipe at flow rates up to 200 gallons per minute. A schematic diagram of the apparatus is seen in Fig. 2.

The total pressure traversing mechanism was designed to provide positive, continuous location of the probe in the tube by means of a micrometer drive and detection of the wall position by means of an electrical circuit consisting of an insulated probe and the electrically conducting wall.

The concentrations of 0.1 and 0.2 percent by weight were run in both pipes, and the 0.05 and 0.4 percent concentrations were run only in the 0.65 and 1.43 - inch pipes, respectively. Measurements of pressure drop per unit length of pipe were made for all concentrations over the full range of possible flow rates. In addition, velocity profile measurements were made at representative flow rates within the range of the pressure drop measurements.

Results

The results of the viscometer tests with the various concentrations of J-2P are shown in Fig. 3, where the shear stress is plotted versus shear rate, logarithmically, over several cycles. The curves drawn through the data make up a family, shifting from essentially a straight line parallel to that for water for 0.05 percent to a curve approaching the slope of the curve for water at either extreme of shear rate for 0.4 percent. Thus, the curve for 0.05 percent shows no variation of viscosity with shear rate, while the curves for 0.1, 0.2, and 0.4 percent are progressively more shear-thinning for increasing concentration. Also included for purposes of comparison is the curve for a 0.4 percent concentration of CMC-70 in water.[18] It should be noted that the onset of turbulence apparently appears around 10^5 sec^{-1} for the 0.05 and 0.1 percent J-2P solutions indicated by the sudden increase in the shear stress for increasing shear rate past that point.

The power-law relation given by Eq. (1) was matched to the data of Fig. 3 in the region of shear rates between 10^3 and 10^5 sec^{-1}. The constants determined in this manner are listed in Table 1.

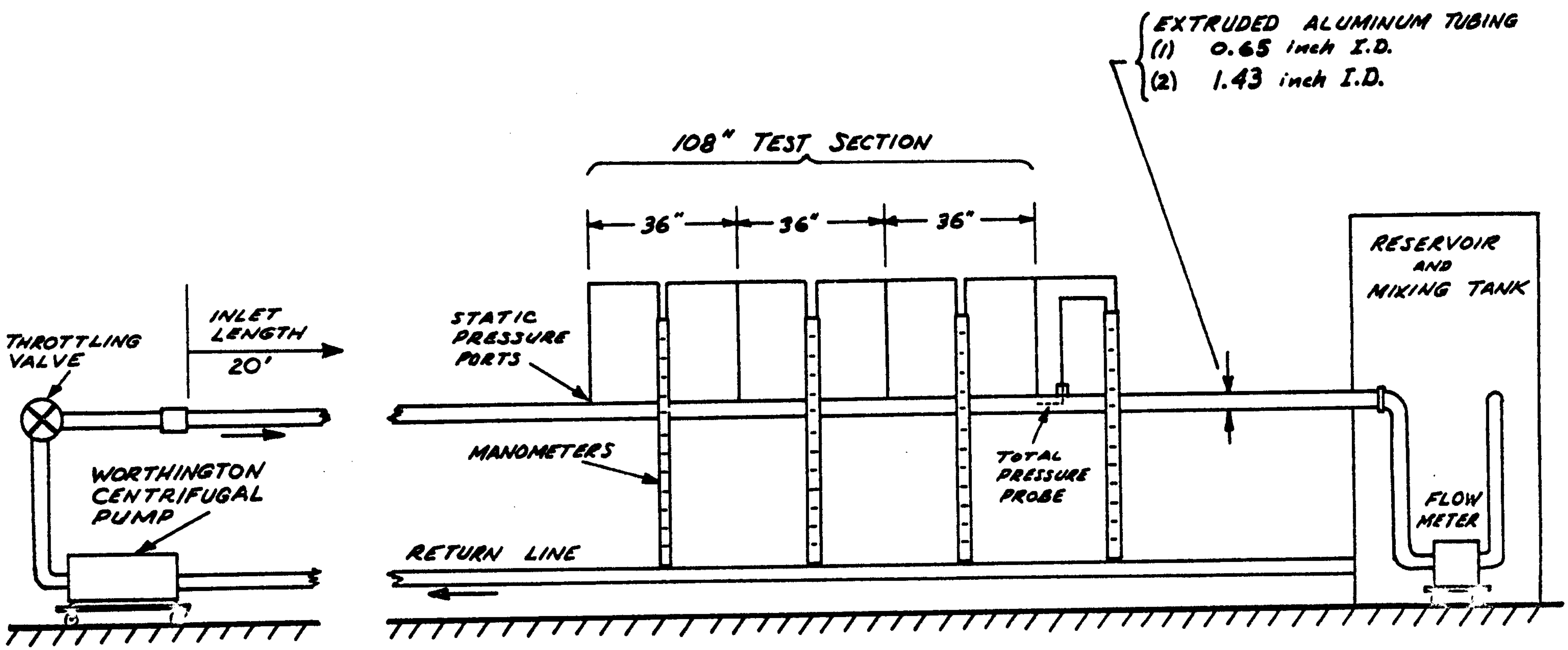


Figure 2 Schematic Diagram of Pipe Flow Apparatus

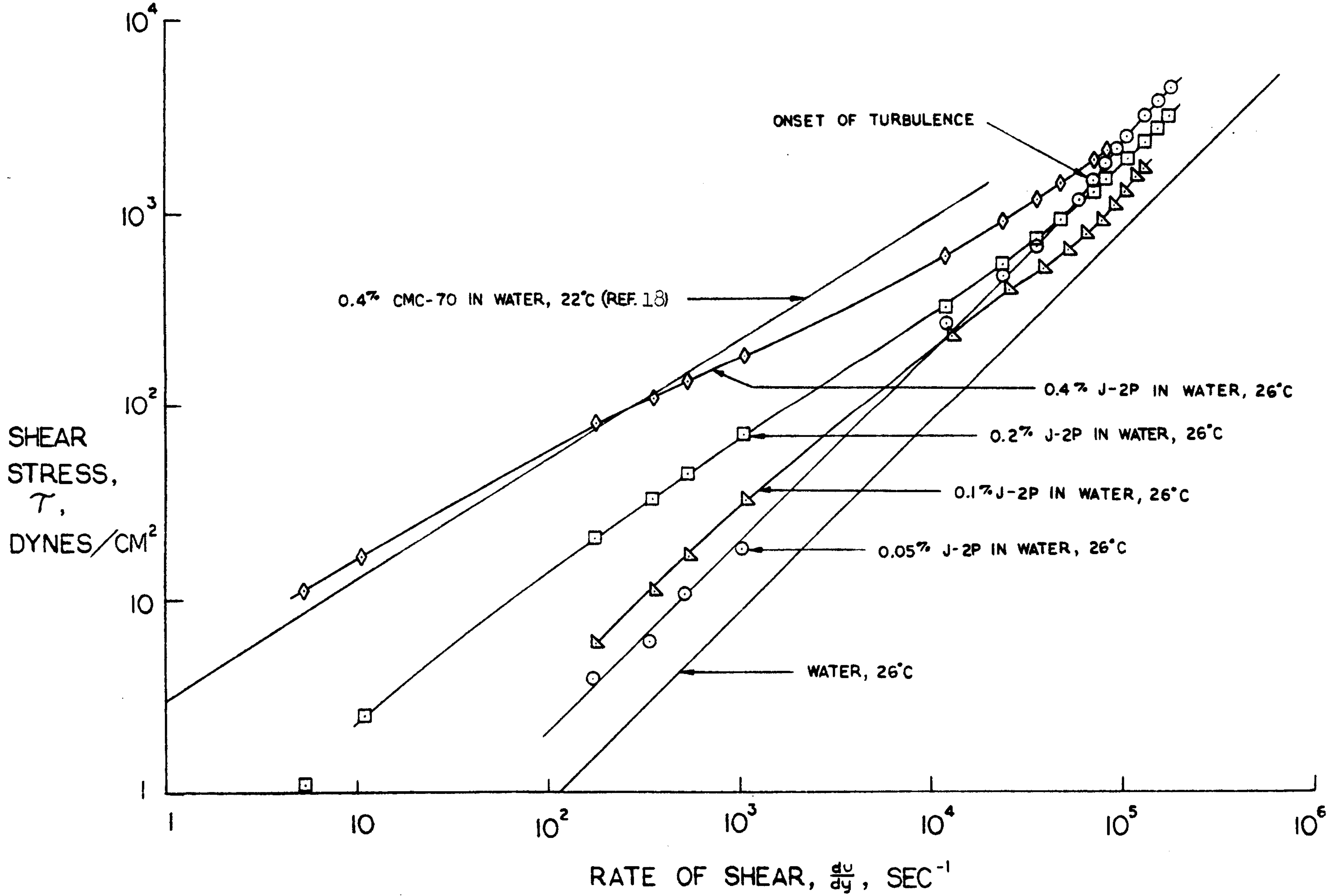


Figure 3 Viscometer Measurements for Several Concentrations of J-2P, Compared with CMC-70 Measurements of Merrill[18].

TABLE I

POWER-LAW CONSTANTS

Fluid	n	a $\left(\frac{\text{gram}}{\text{cm sec}^{2-n}}\right)$
0.05% J-2P	1.0	0.022
0.1 % J-2P	0.81	0.12
0.2 % J-2P	0.71	0.48
0.4 % J-2P	0.56	3.38
0.04% CMC-70	0.62	3.03

The values in Table 1 were used to reduce the data from the turbulent pipe flow experiments, since the maximum (wall) shear rate values of the experiments fell within the range of shear rates used to calculate the constants.

The basic results of the measurements of pressure drop due to friction for turbulent flow are shown in Figs. 4 and 5. Friction factor, defined by

$$f = \frac{2\,\tau_o}{\rho\,\hat{u}^2} \quad , \tag{18}$$

was plotted versus the equivalent Reynolds number for water, Re_s, defined by

$$Re_s = \frac{\rho\,\hat{u}\,D}{\mu_{water}} \quad . \tag{19}$$

τ_o was determined from the measured pressure drop per unit length, assuming fully developed flow, by means of the force balance:

$$\tau_o = \frac{R}{2}\,\frac{dp}{dx} \quad .$$

Data are shown for the flow of pure water and concentrations of 0.05, 0.1, 0.2, and 0.4 percent J-2P in the two pipes. Also shown are the derived expression and empirical formulation for fully-developed laminar and turbulent

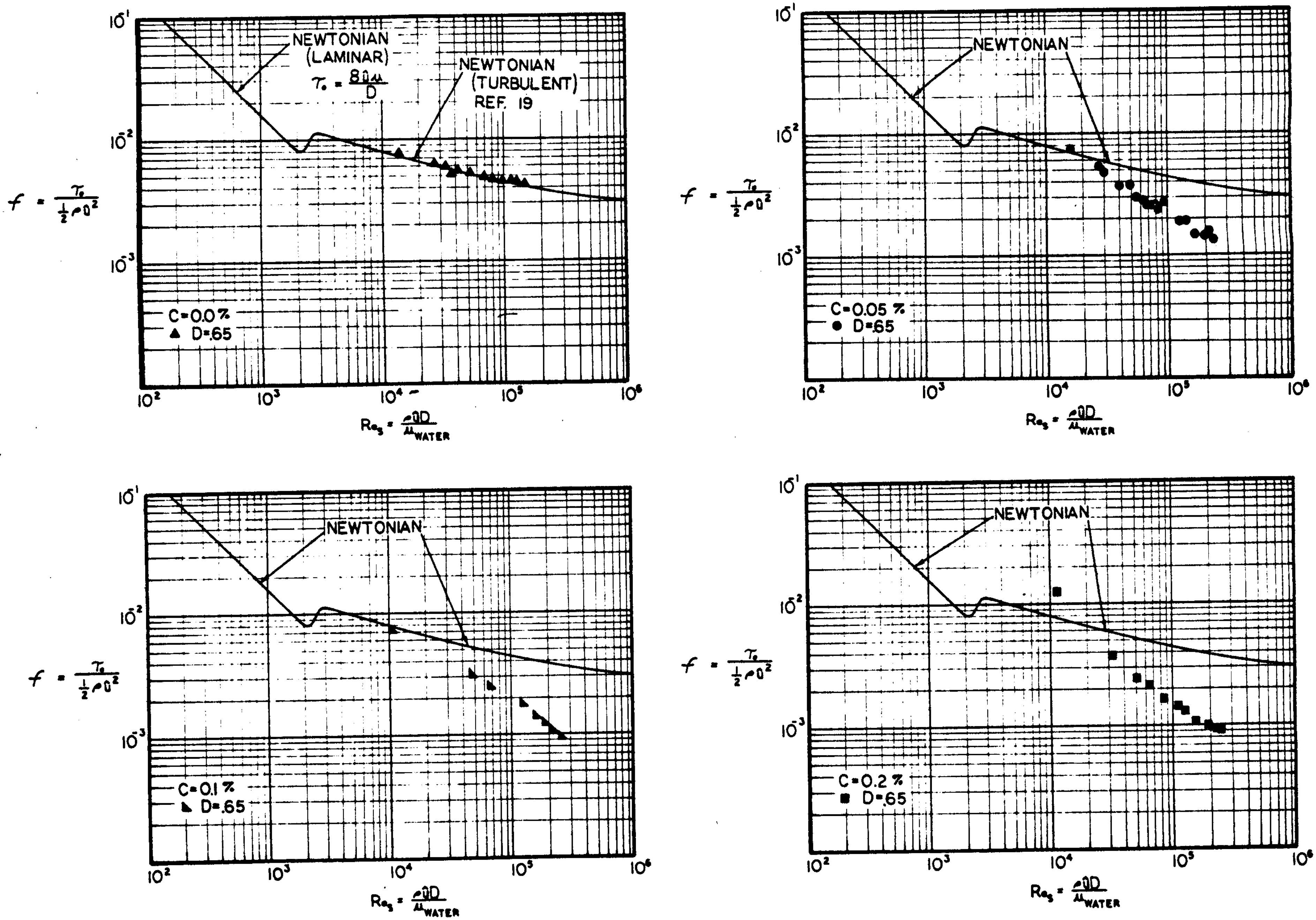


Figure 4 Friction Factors for Several Concentrations of J-2P, Plotted Versus Reynolds Number of the Solvent, for D = 0.65 Inches.

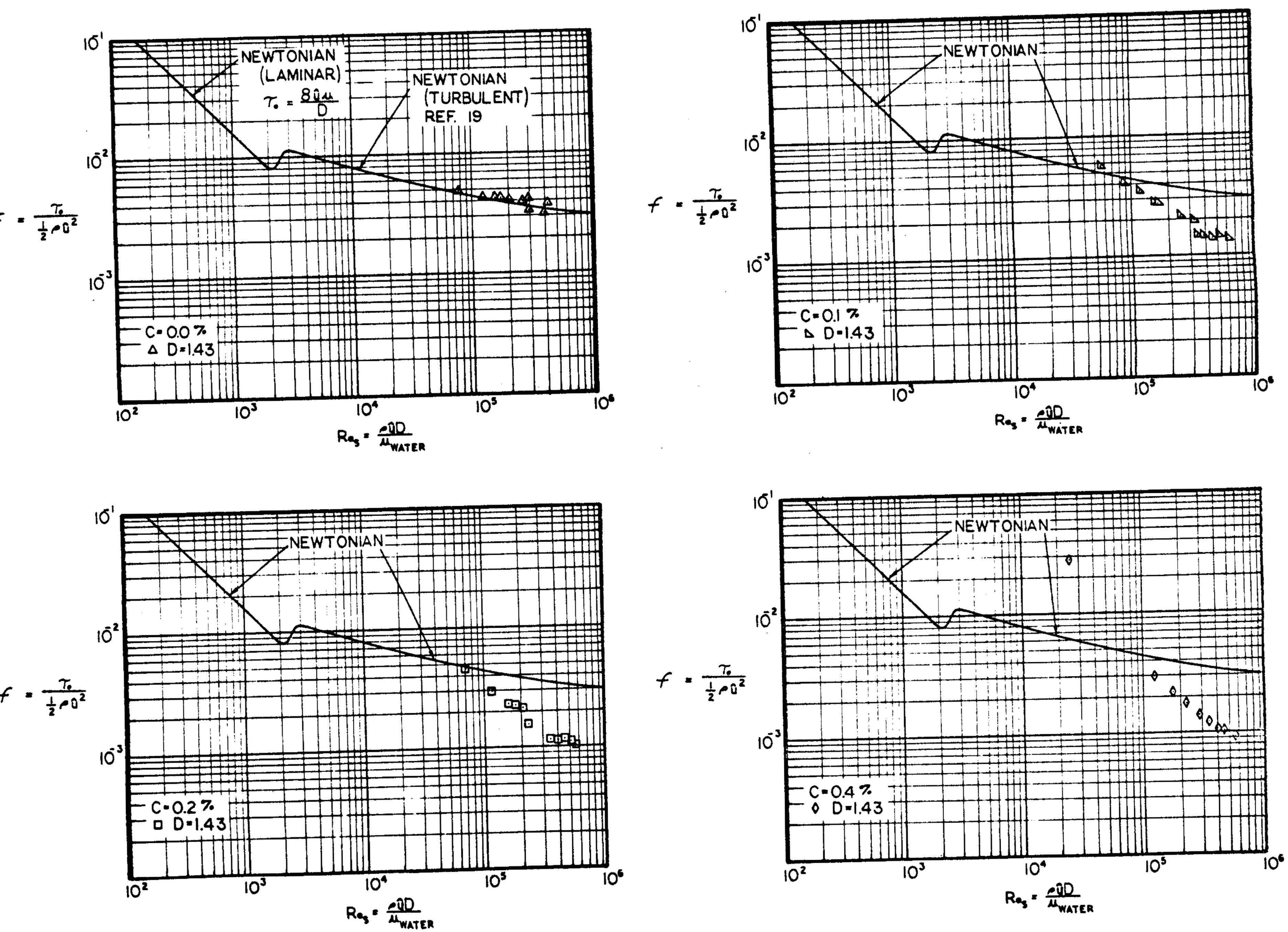


Figure 5 Friction Factors for Several Concentrations of J-2P, Plotted Versus Reynolds Number of the Solvent, for D = 1.43 Inches.

flow of Newtonian fluids through a pipe[19]. These are given by

$$f = \frac{16}{Re_s} \qquad \text{(laminar)} \tag{20a}$$

$$\frac{1}{f^{\frac{1}{2}}} = 4.0 \log_{10} (Re_s\, f^{\frac{1}{2}}) -0.40 \quad \text{(turbulent)} \tag{20b}$$

The friction factors for the polymer solutions plotted in these coordinates are seen to fall below the friction factors for Newtonian fluids, with two exceptions which will be shown below to represent laminar and transitional flow.

A relation for laminar flow of purely viscous power-law fluids analagous to Eq. (20a) for Newtonian fluids can be derived. The resultant expression for friction factor is

$$f = \frac{16\ a}{\rho\ u^{2-n}\ D^{n}} \quad \frac{[\ 2\ (3 + \frac{1}{n})]^{n}}{8} \quad , \tag{21}$$

for which Eq. (20a) is seen to be a special case, for $n = 1$ and $a = \mu$. Eq. (21) can be rewritten by defining new variables as

$$f = \frac{16}{Re_n\ \psi_n} \tag{22}$$

where Re_n and ψ_n are defined from Eq. (21).

The friction factor data of Figs. 4 and 5 have been plotted in terms of $Re_n\ \psi_n$ in order to obtain a better comparison with Newtonian fluids in both the laminar and turbulent regions. This is shown in Fig. 6. It appears that the variables of Eq. (22) do correlate the data in the laminar region, as evidenced by the trends of the turbulent data and from the lone data point for 0.4 percent J-2P.

Here again, the friction factors for $Re_n\ \psi_n$ values which should certainly represent turbulent flow for Newtonian fluids are much lower than those for Newtonian fluids. Something which is pointed out by plotting the data

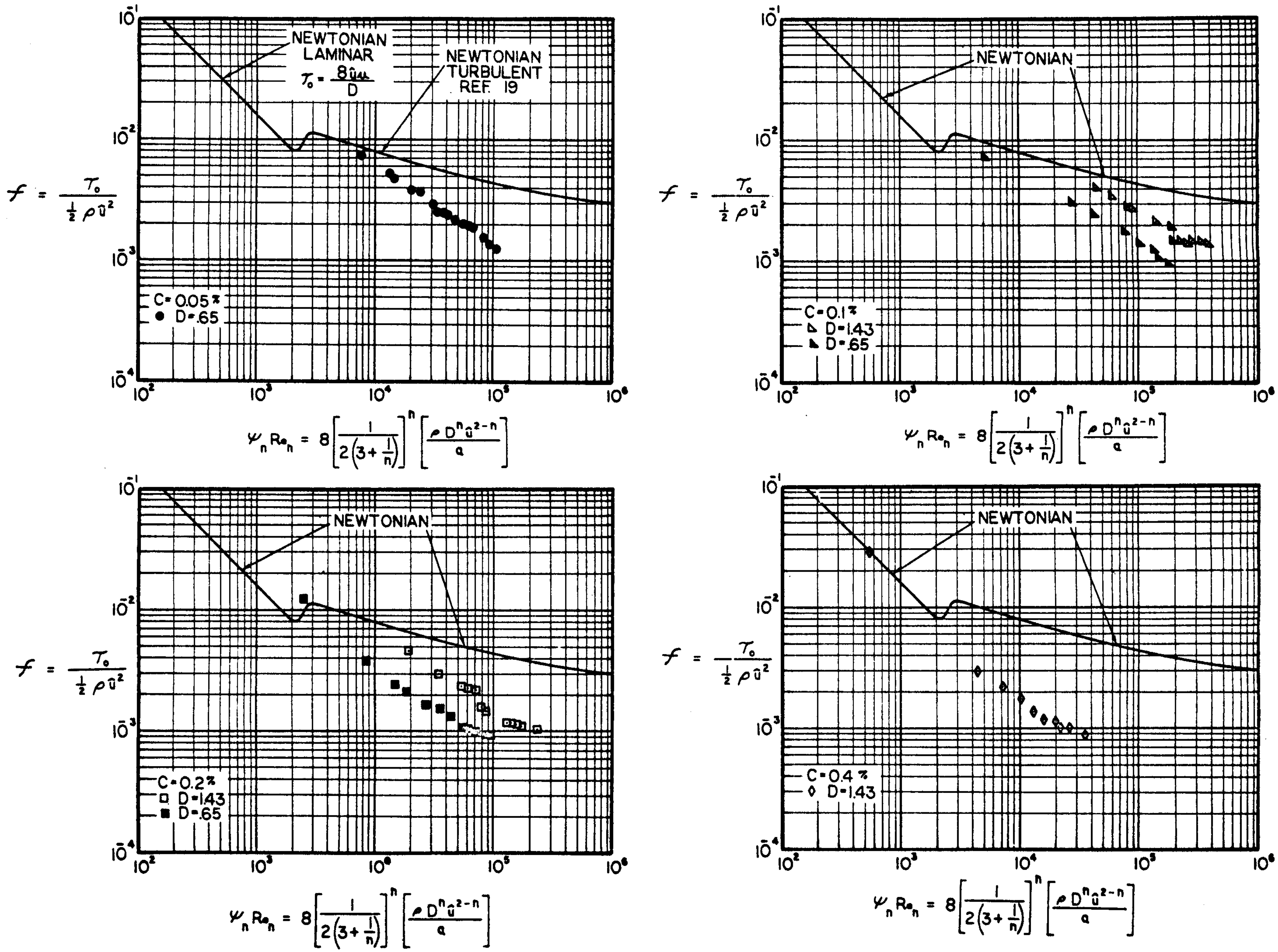


Figure 6 Friction Factors for Several Concentrations of J-2P, Plotted Versus the Purely - Viscous Power Law Reynolds Number for D = 0.65 and D = 1.43 Inches.

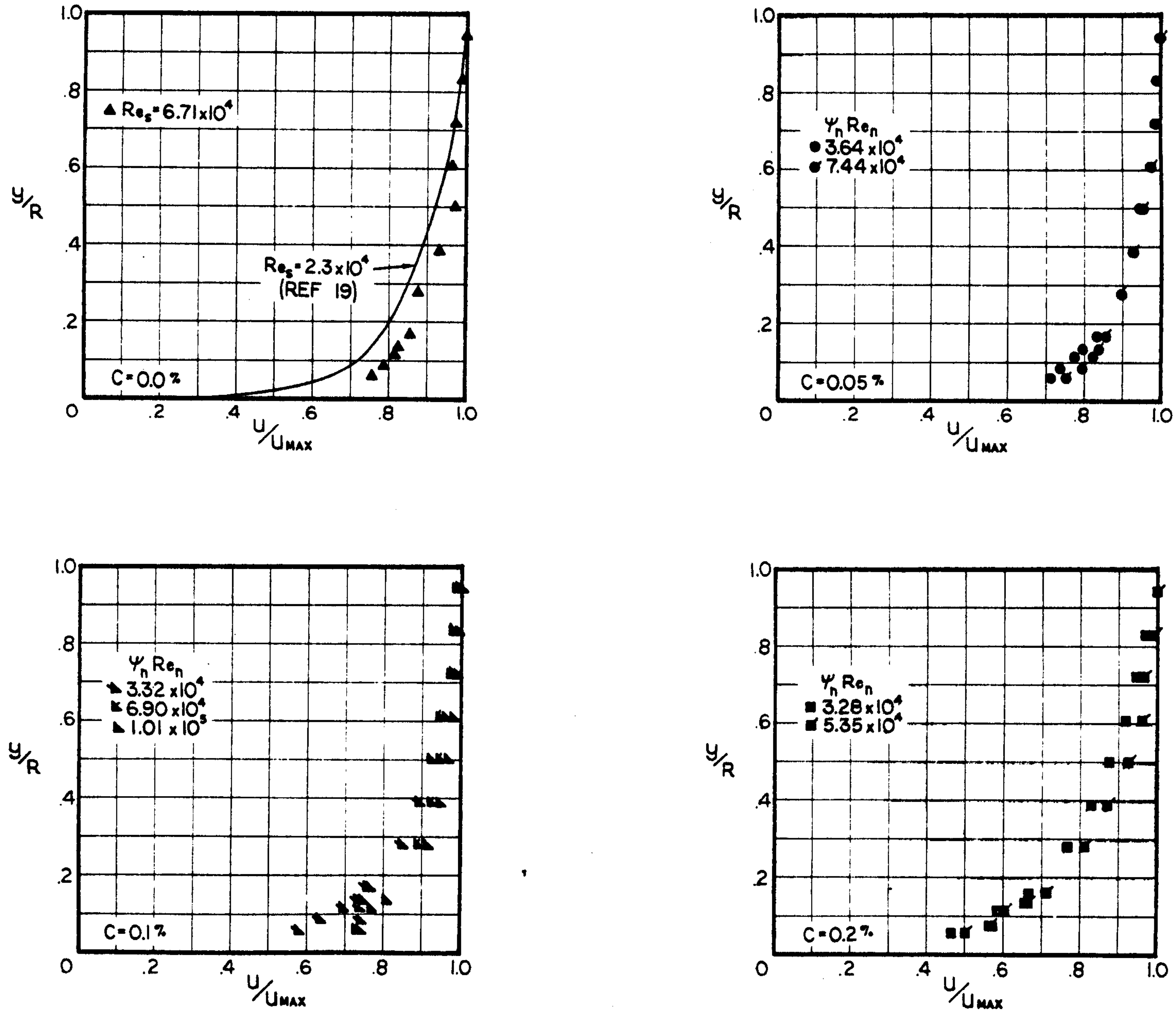


Figure 7 Velocity Profiles for Several Concentrations of J-2P, D = 0.65 Inches.

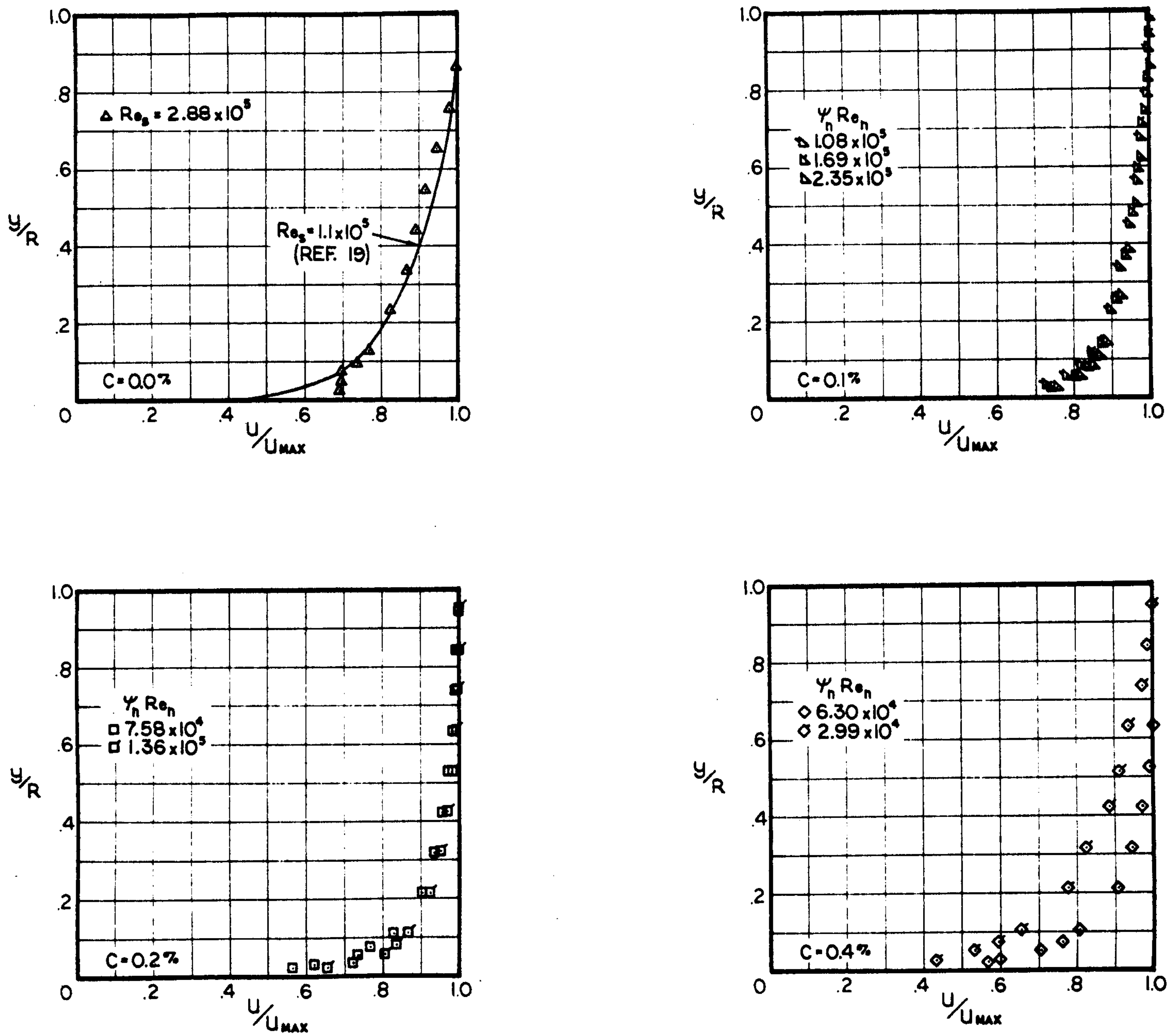


Figure 8 Velocity Profiles for Several Concentrations of J-2P, D = 1.43 Inches.

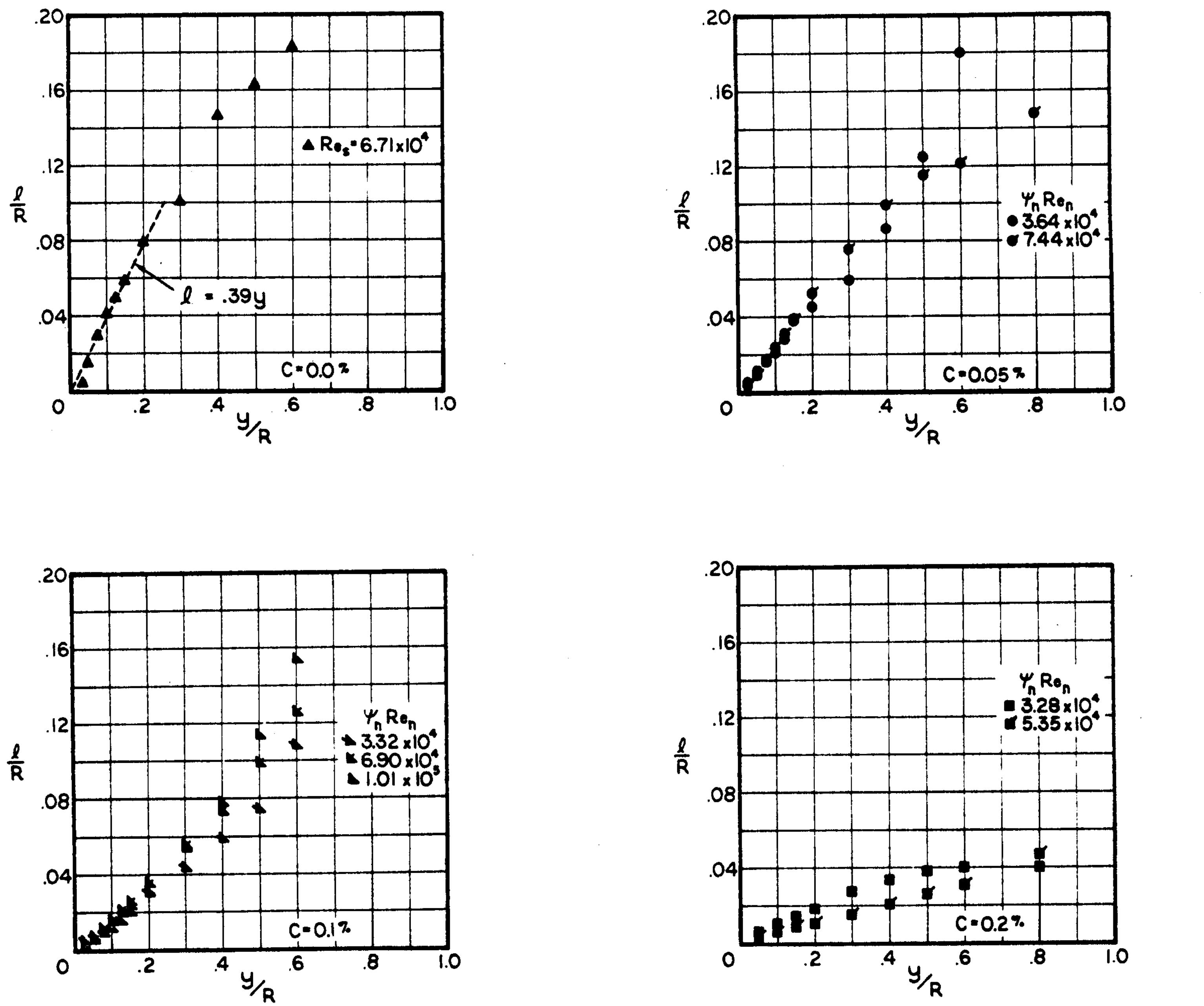


Figure 9 Results of Mixing Length Calculations for D = 0.65 Inches.

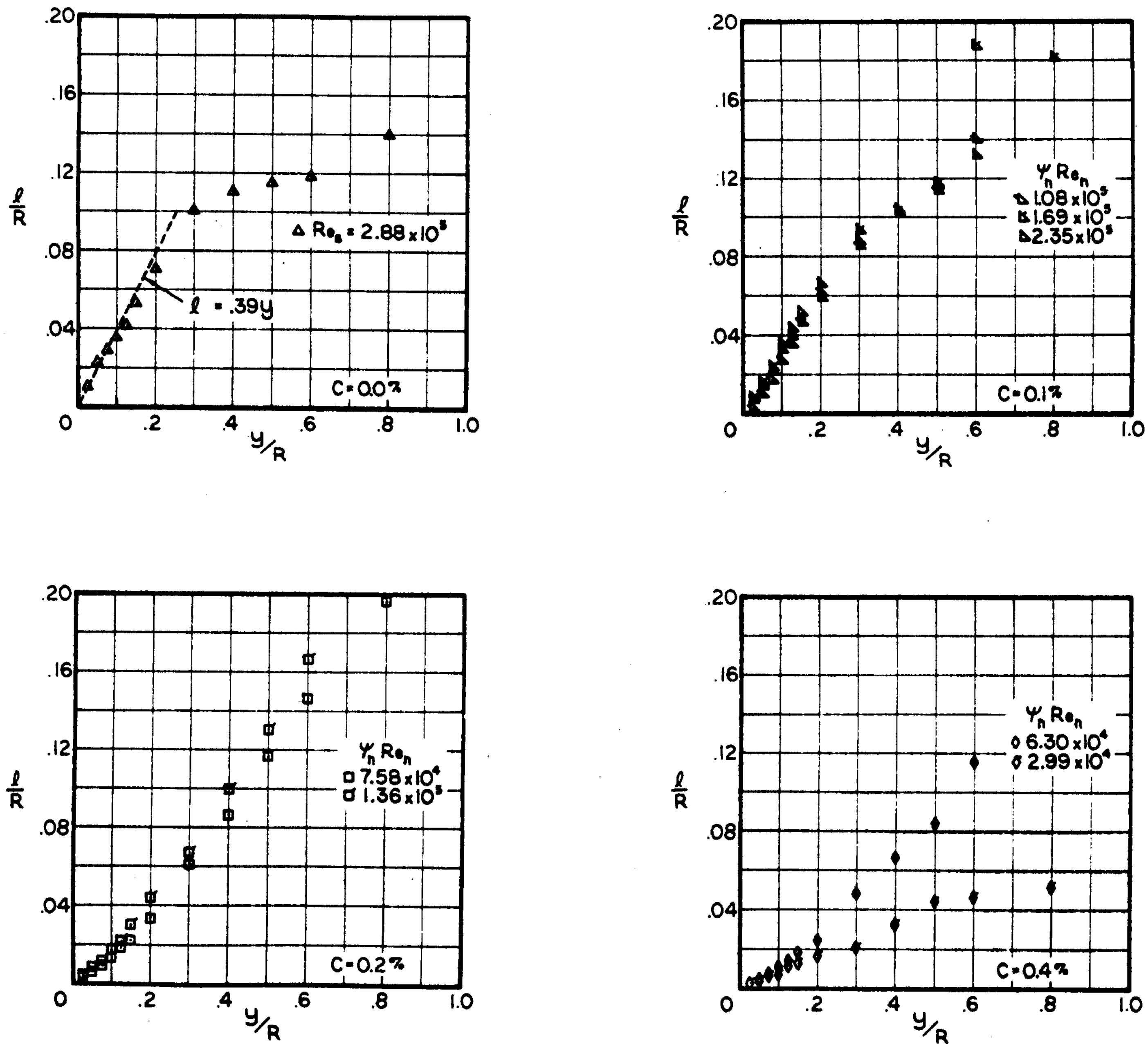


Figure 10 Results of Mixing Length Calculations for D = 1.43 Inches.

in this manner is the effect of diameter on the correlation. For the concentrations run in both pipe sizes (0.1 and 0.2 percent) the data for the two pipe sizes are seen to lie on separate curves.

Another interesting point to be seen in Fig. 6 is that there is a trend with higher concentration (more shear thinning and presumably more elasticity) and smaller pipe diameter for the friction factor data to follow an extension of the theoretical laminar curve to very low values of friction factor.

The basic velocity profile data shown in Figs. 7 and 8 were used to determine the variation of mixing length, ℓ, as defined in Eq. (3), with distance from the wall. The gross assumption of Prandtl[15] that the shear stress through the shear layer is equal to the wall shear stress, τ_o was applied. Therefore, for a given profile, the measurement of τ_o and the calculation from the measured profile of du/dy as a function of y was used in Eq. (3) to calculate ℓ as a function of y. The results of these calculations are shown in Figs. 9 and 10. As for Newtonian fluids, the variation of ℓ vs. y was seen to approach an asymptote near the wall and the approximation of Eq. (5) was applied to determine the mixing length coefficient, k, for each profile. In each case the mixing length constants determined in this way were found to be smaller than the universal constant for Newtonian fluids. For a given fluid (concentration of J-2P) k appears to vary with pipe diameter and possibly with Reynolds number. It should be noted that the calculation of k carries an estimated uncertainty of plus or minus 15 percent of the stated value. This is due to the compounded uncertainty incurred by determining the local slope of a curve drawn through experimental data points and squaring the result, as required by Eq. (3). Even with this uncertainty in mind, however, it is possible to say that k decreases with decreasing pipe diameter, for the same Reynolds number range. The trend of k with Re_n is not clear as it does lie within the stated uncertainty in most cases and the data are not extensive enough to define a trend.

Figs. 11 through 16 present the data for all of the fluids used in the experiments, and the data for another elastico-viscous fluid taken from the literature[5], in terms of the variables defined by Eq. (8). Also shown in these figures are the equations derived for the viscous sublayer and the

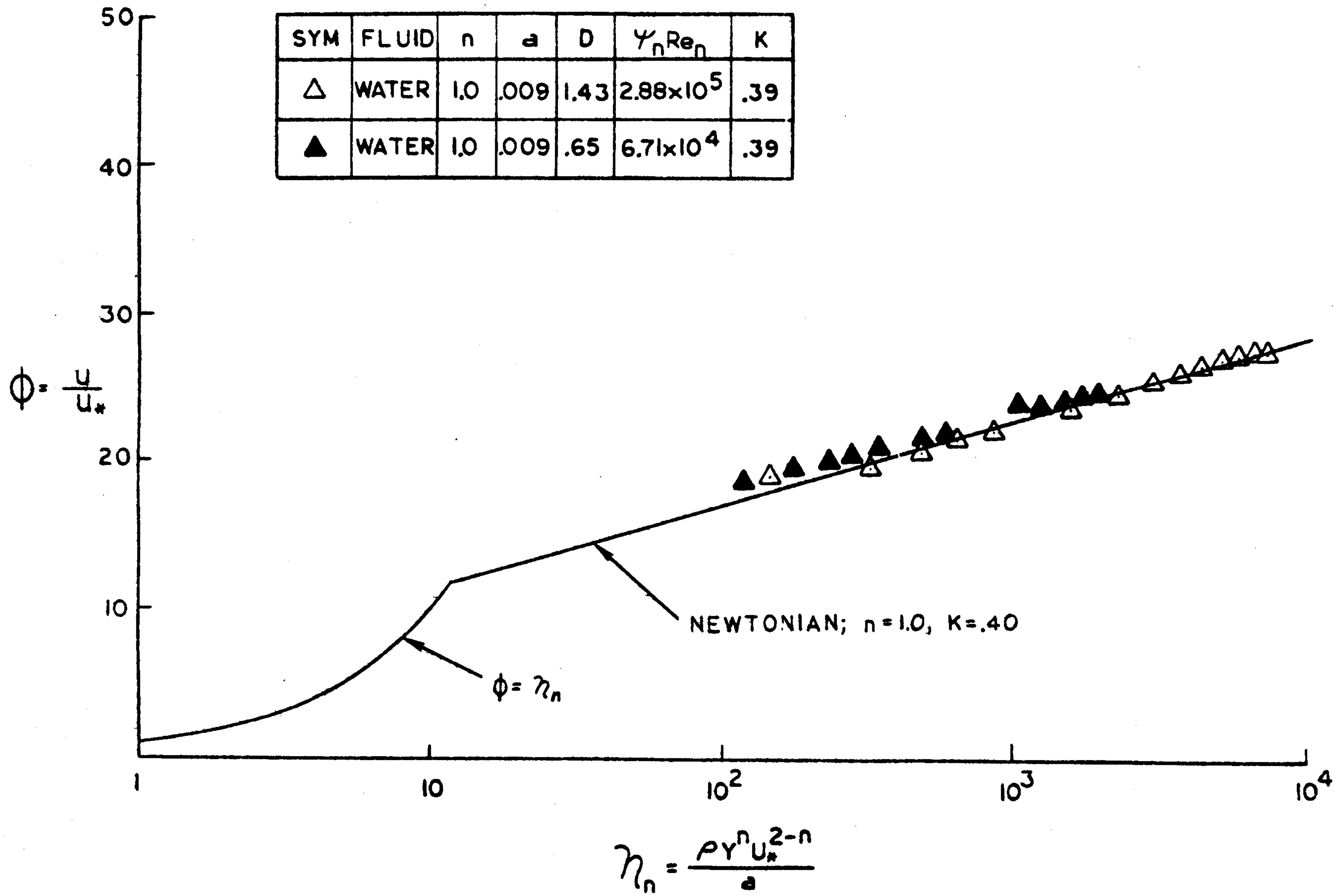


FIGURE 11 GENERALIZED VELOCITY PROFILES FOR WATER

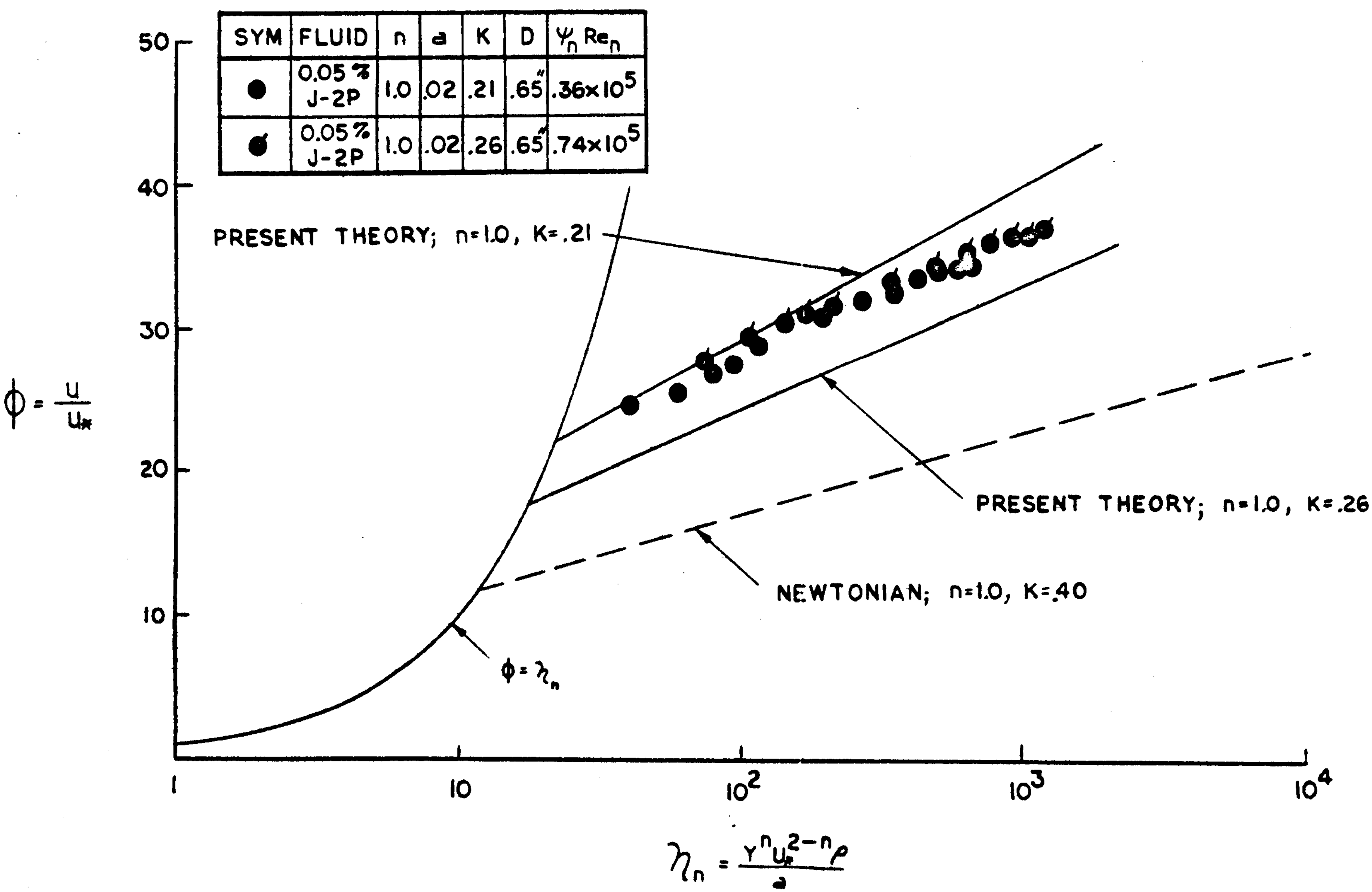


Figure 12 Generalized Velocity Profiles for 0.05% J-2P in Water.

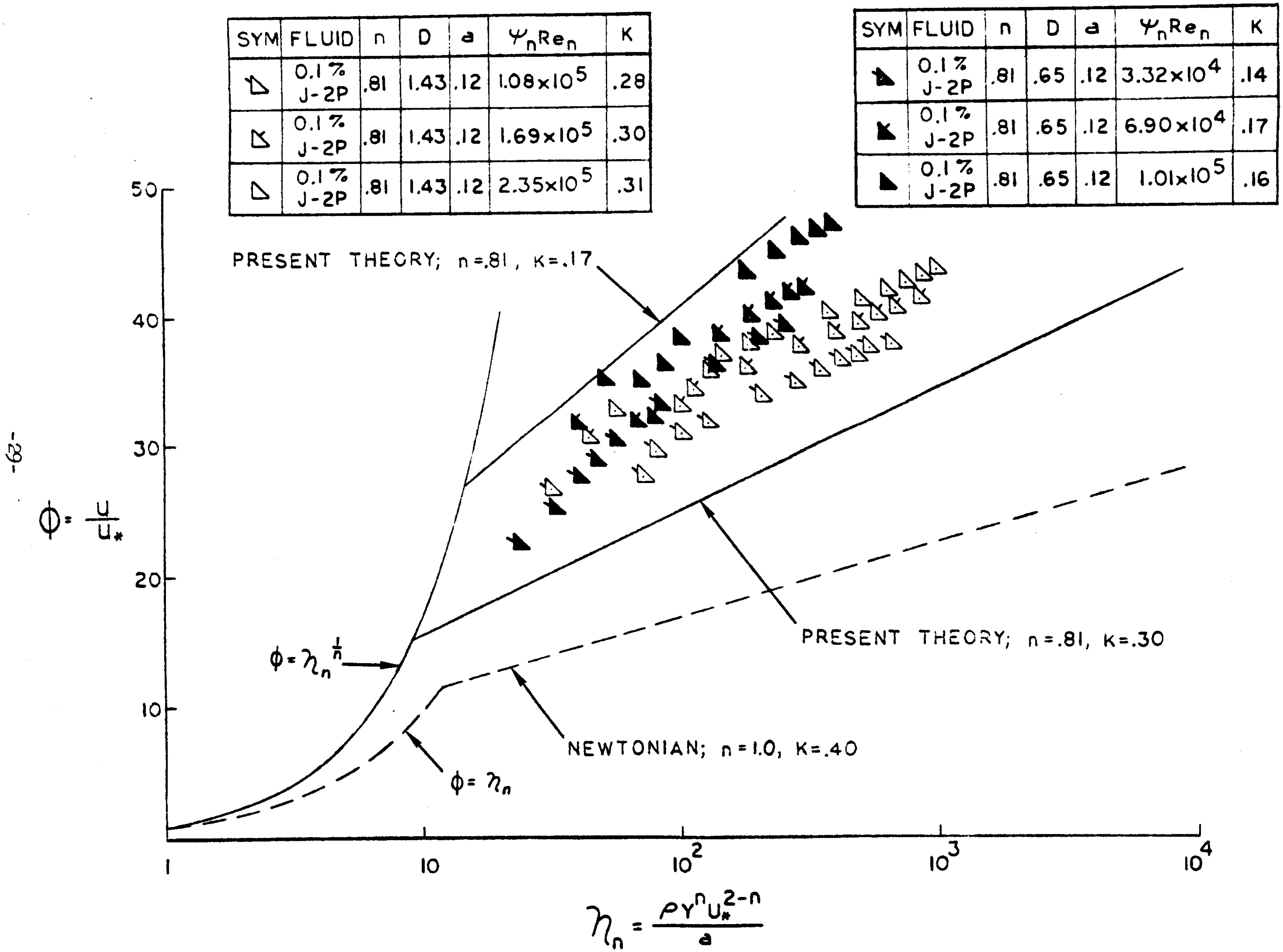
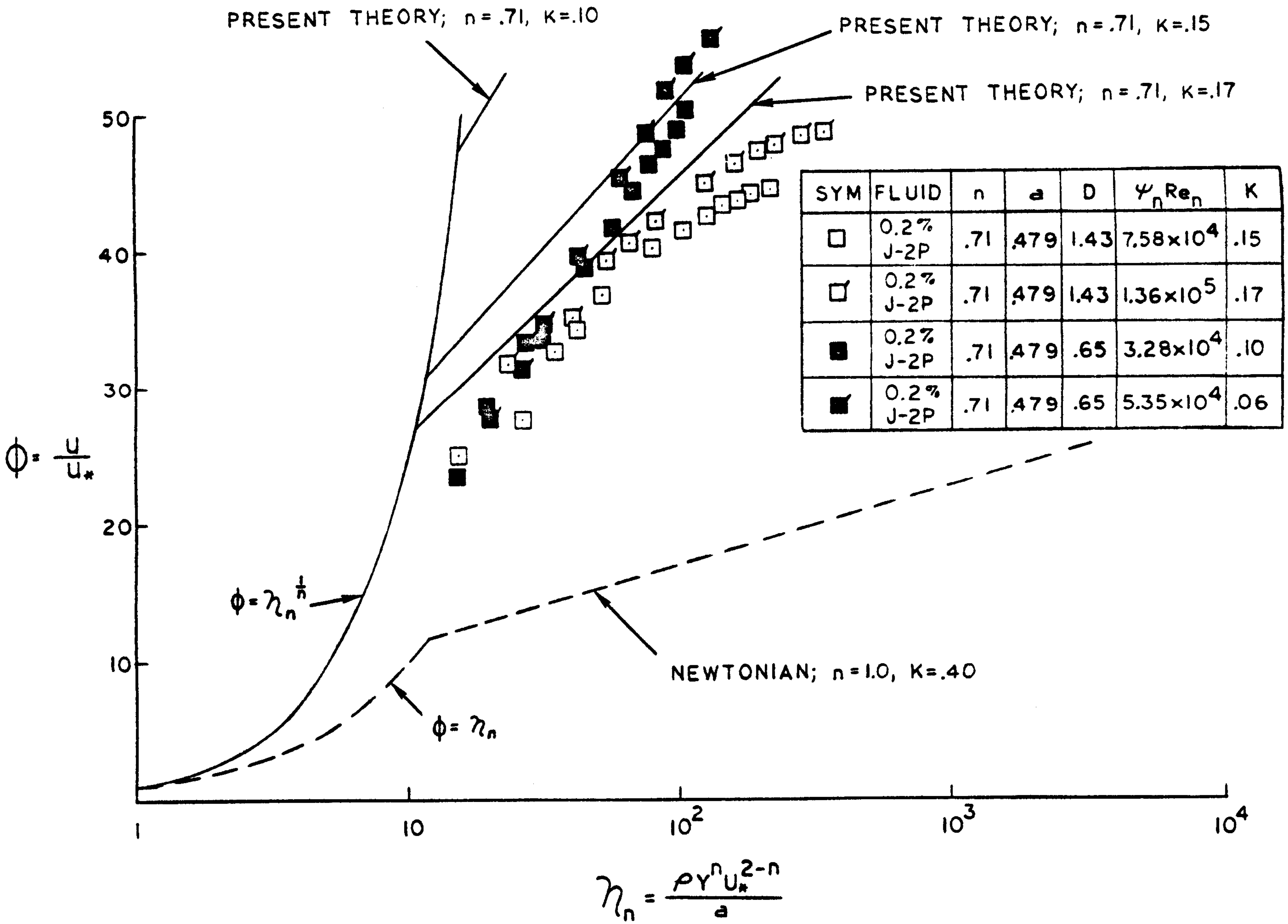
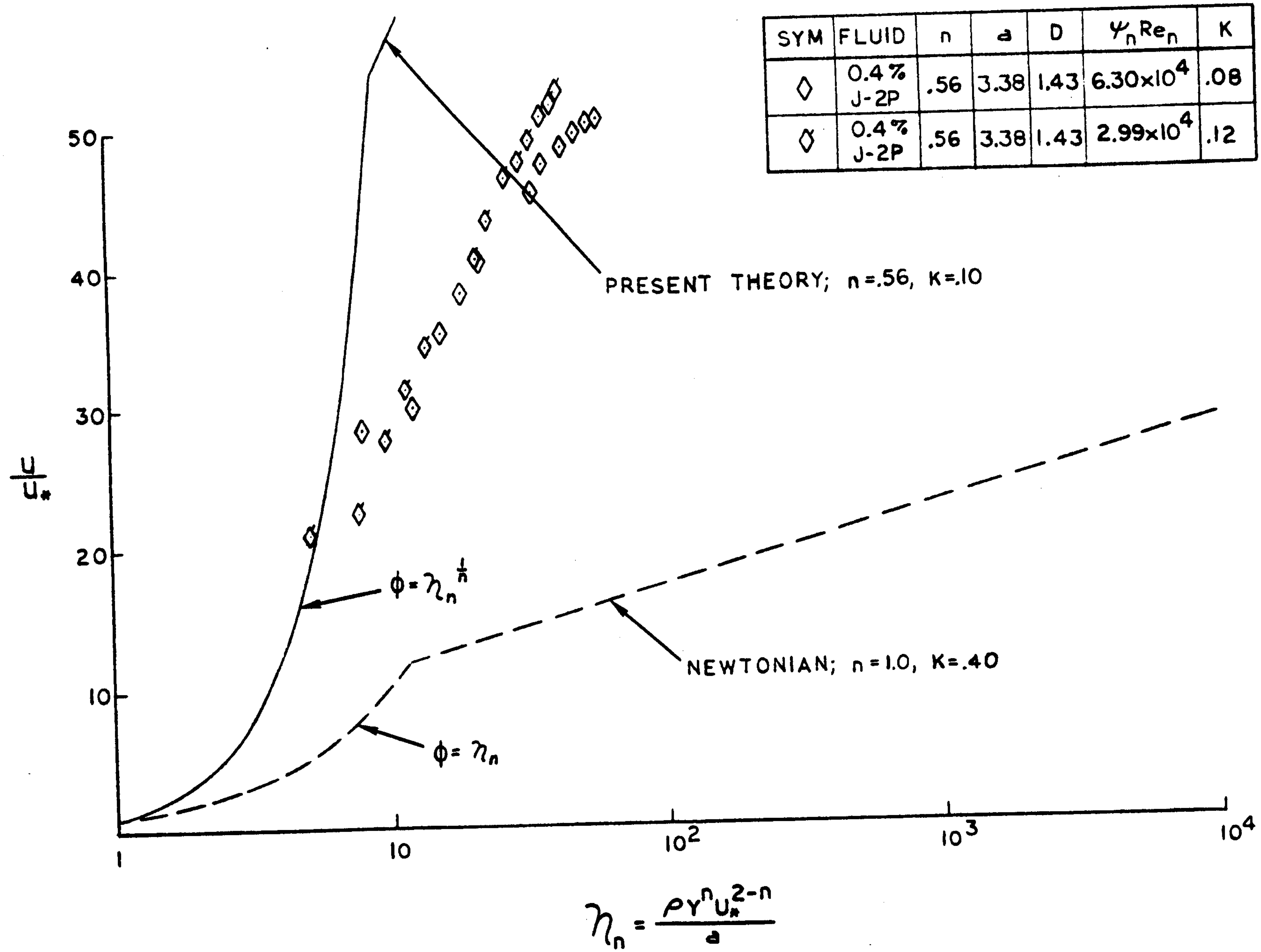


Figure 13 Generalized Velocity Profiles for 0.1% J-2P in Water.



SYM	FLUID	n	a	D	$\psi_n Re_n$	K
□	0.2% J-2P	.71	.479	1.43	7.58×10^4	.15
□	0.2% J-2P	.71	.479	1.43	1.36×10^5	.17
■	0.2% J-2P	.71	.479	.65	3.28×10^4	.10
■	0.2% J-2P	.71	.479	.65	5.35×10^4	.06

Figure 14 Generalized Velocity Profiles for 0.2% J-2P in Water.



SYM	FLUID	n	a	D	$\psi_n Re_n$	K
◊	0.4% J-2P	.56	3.38	1.43	6.30×10^4	.08
◊̸	0.4% J-2P	.56	3.38	1.43	2.99×10^4	.12

Figure 15 Generalized Velocity Profiles for 0.4% J-2P in Water.

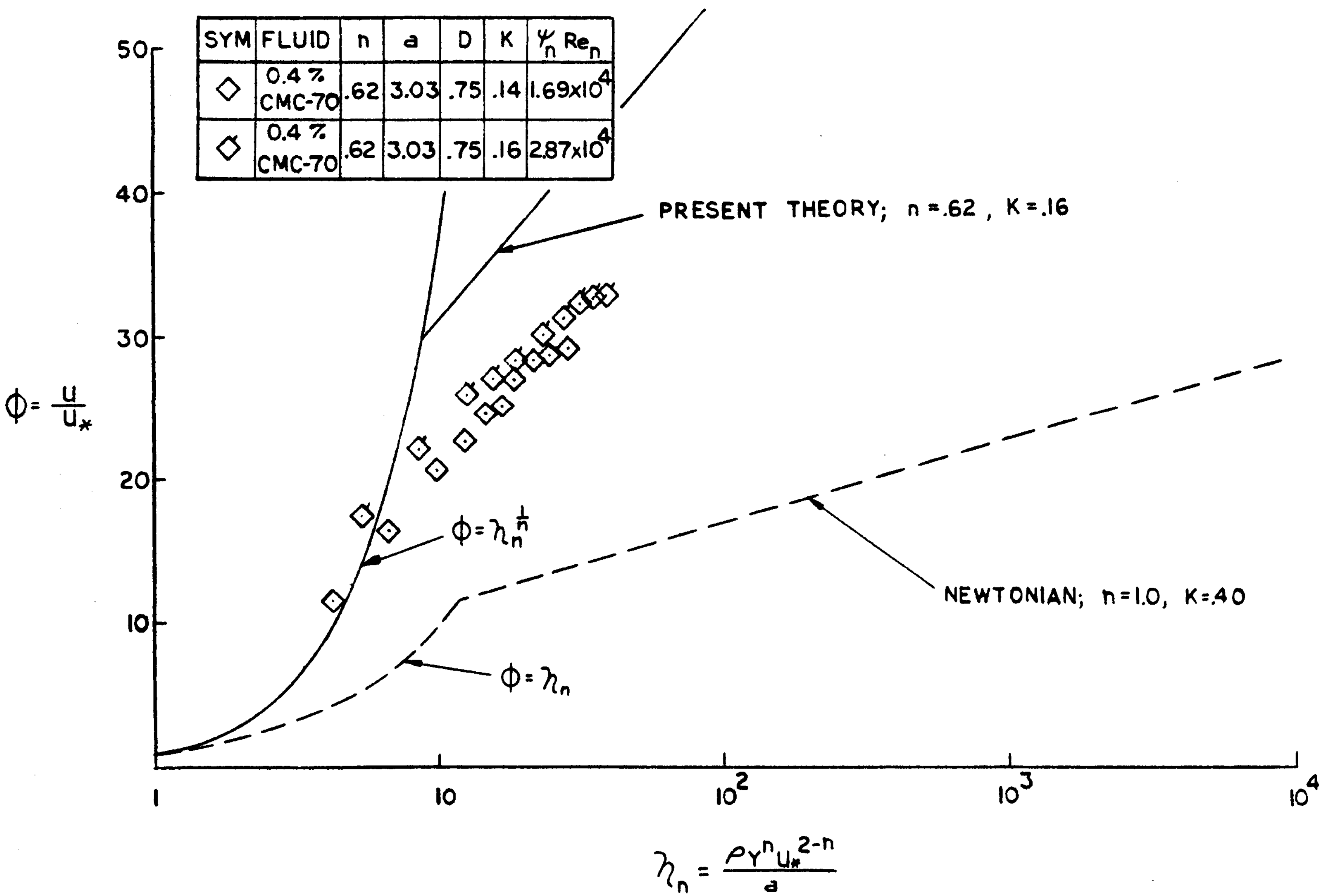


Figure 16 Generalized Velocity Profiles for 0.4% CMC-70 in Water.

turbulent region, Eqs. (8) and (16), respectively. Appropriate experimental values of n, a and k were used in the calculations. The values of k chosen for the calculations were considered typical and in some cases two values of k were chosen to show the spread in the experimental values.

The data for water shown in Fig. 11 are seen to be in good agreement with the semi-logarithmic relation for Newtonian fluids given by Eq. (17) with no data indicated in the sublayer region.

The velocity profiles for 0.05 percent J-2P, seen in Fig. 12, is one of the most interesting since the viscometer measurements for that fluid indicated no shear-thinning; i.e., n = 1. The data are seen to compare reasonably well with Eq. (16); and the height of the sublayer, defined by the intersection of the curves for the viscous and turbulent regions, is seen to be larger by two or three times than for Newtonian fluids. Again the data appear to lie wholly in the turbulent region.

Figures 13 and 14 present velocity profiles for 0.1 and 0.2 percent J-2P solutions. Typical values of k for each of the two pipe diameters were chosen to calculate Eq. (16). The data are seen to lie between the curves for the turbulent region for the 0.1 percent data and generally below the turbulent curves for the 0.2 percent data. In general, however, both sets of data and both sets of semi-logarithmic curves indicate a thicker viscous sublayer, calculated from the intersection of Eqs. (8) and (16), than for Newtonian fluids. An increase in sublayer thickness with increasing Reynolds number (or velocity) also is shown by the data, indicating an increased stability, although not to the extent predicted by the analysis. The slopes of the profile in the turbulent region is predicted reasonably well by the analysis and the experimental mixing length coefficient.

Figures 15 and 16 present data for the most viscous and presumably most elastic fluids, 0.4 percent J-2P and 0.4 percent CMC-70. These data show reasonable agreement with, and a trend toward, the expression for the viscous sublayer as given by Eq. (8). Here again the data show increased stability by a thickening of the sublayer, although not to the extent predicted, and the slope of the turbulent profile agrees with the slope predicted.

DISCUSSION AND CONCLUSIONS

The results of the analysis and the experiments with turbulent shear flow of elastico-viscous fluids can now be discussed. It should be pointed out again that the experimental data were analyzed in terms of concepts proved successful for Newtonian fluids, but that the elastic properties of the fluids were not used explicitly. This approach first enabled a comparison to be made between turbulent flow of Newtonian and elastico-viscous fluids, and then made it possible to draw conclusions about the details of the flow.

The Prandtl mixing length concept made it possible to relate the data from the turbulent flow experiments to the effects of elasticity; however, the resulting analysis will be of practical value in predicting the velocity profile only if the relation between the elastic properties and the mixing length constant is found. This is evident from the experimentally observed variation of the mixing length constant.

With this limitation in mind, it can be stated that the velocity profiles predicted for the two assumed regions of flow in the turbulent shear layer from mixing length and stability considerations agree reasonably well with the experimental data. The velocity profiles represent generalizations of expressions for Newtonian and purely viscous non-Newtonian fluids, and agree with data for Newtonian fluids.

The experimental data appear to show an over-all increased stability to production of turbulence for the elastico-viscous fluids. This is shown both in the friction factor plots, where a more gradual transition from laminar to turbulent flow is shown, particularly for the higher concentrations of the polymer (and presumably greater elasticity); and in the velocity profile plots, where the transition region between the sublayer and the fully turbulent region may become a large part of the total shear layer.

Perhaps one of the most interesting results of the experiments is the fact that the viscous sublayer, as defined in the analysis, was thicker than for Newtonian or purely viscous non-Newtonian fluids for all of the elastico-viscous velocity profiles examined. According to the analysis, this is a result of increased stability as calculated from the mixing length constant.

The range of variables of the experiments was such that no definitive statements can be made regarding variations of the results with Reynolds number. In general, it appears that the frictional drag reduction and the viscous sublayer thickness increase with Reynolds numbers. This coupling between the drag reduction and sublayer thickness also appears as the effect of pipe diameter, which is noted in both types of data. The effect of diameter, while not understood at this time, is apparently an additional scale effect due to the presence of the constraining wall in a mixing process modified by the fluid elasticity. The fact that the mixing length constant shows an effect of diameter is further indication of this additional scale effect on the mixing itself.

The following conclusions can then be summarized regarding the analysis and experiments:

(1) Turbulent shear flow of Newtonian fluid solvents with small concentrations of high molecular weight polymers, presumed to be elastico-viscous fluids, demonstrate a reduction in frictional drag over that of the Newtonian solvents. Further, the drag reduction increases with increasing Reynolds number, although there is a pronounced effect of diameter on the drag results.

(2) A viscous sublayer, always thicker than for purely viscous fluids, lies adjacent to the constraining wall for these fluids.

(3) The variation of velocity with distance from the wall approaches, near the wall, a relation derived for a constant shear stress flow and which is based on the viscous fluid properties only.

(4) The variation of velocity with distance from the wall approaches, far from the wall, a relation based on the Prandtl mixing length concept and a variable mixing length coefficient.

(5) The increase in the thickness of the viscous sublayer noted in the data is predicted by a stability criterion stating that turbulence occurs when the ratio of turbulent to laminar shear stresses reaches a universally constant value. The predicted values of the viscous sublayer thickness do not agree with the data for the highest polymer concentrations (and elasticity). This lack of agreement is thought to be due to one or both of two possible factors:

(a) An independent effect of pipe diameter, or the proximity of the opposite constraining walls, for fluids with higher elasticity and stability.

(b) A dependence of the transition criterion on the non-linear shear stress relation, not taken into account in the present criterion.

These possibilities are being investigated through further analysis of the present data, as well as further experiments.

ACKNOWLEDGMENTS

The author would like to acknowledge the help and encouragement of Dr. F. W. Fenter and Dr. John Harkness in the many discussions of this problem. The work of Mr. J. G. Spangler and Mr. W. D. Ernst in obtaining the viscosity data also contributed to this study. The investigation was begun as a part of the Ling-Temco-Vought, Inc. independent research and development program and is presently partially supported by the National Aeronautics and Space Administration under Contract No. NASw-729.

SYMBOLS

a	non-Newtonian fluid property, defined by Eq. (1)
A	virtual kinematic viscosity, defined by Eq. (2)
C	concentration, percent by weight
D	pipe diameter
f	friction factor, defined by Eq. (18)
k	mixing length constant, defined by Eq. (5)
ℓ	mixing length, defined by Eq. (3)
n	non-Newtonian fluid index, defined by Eq. (1)
R	pipe radius
Re_s	solvent Reynolds number defined by Eq. (19)
Re_n	purely viscous Reynolds number, defined by Eqs. (21), (22)
u	velocity in the x-direction
u_*	friction velocity $(\tau_o/\rho)^{\frac{1}{2}}$
v	velocity in the y-direction
w	velocity in the z-direction
x	streamwise coordinate
y	normal coordinate, distance from the wall
z	coordinate normal to the x-y plane
ϵ	virtual viscosity, defined by Eq. (2)
τ	shear stress
ρ	density
ϕ	non-dimensional velocity, defined by Eq. (8)
η_n	non-dimensional normal coordinate, defined by Eq. (8)
ψ_n	constant, defined by Eqs. (21), (22)

Subscripts

L	edge of the viscous sublayer
o	wall
t	turbulent

Superscripts

$\bar{}$	time averaged quantity
$\hat{}$	spacially averaged quantity

REFERENCES

1. Toms, B. A., Proceedings International Rheological Congress, Holland (North Holland Publishing Co., 1949).

2. Savins, J. G., "Some Comments on Pumping Requirements for non-Newtonian Fluids," Journal of the Institute of Petroleum, Vol. 47, No. 454 (1961).

3. Dodge, D. W., and Metzner, A. B., "Turbulent Flow of Non-Newtonian Systems", J. Amer. Inst. Chem. Engrs., Vol. 5, No. 2, (pp. 189-203 (1959)).

4. Metzner, A. B., Houghton, W. T., Sailor, R. A., and White, J. L., "A Method for the Measurement of Normal Stresses in Simple Shearing Flow", Transactions of the Society of Rheology, V, pp. 133-147 (1961).

5. Shaver, R. G., and Merrill, E. W., "Turbulent Flow of Pseudoplastic Solutions in Straight Cylindrical Tubes", J. Amer. Inst. Chem. Engrs., Vol. 5, No. 2. pp. 181 - 187 (1959).

6. Ousterhout, R. S., and Hall, C. D. Jr., "Reduction of Friction Loss in Fracturing Operations," Journal of Petroleum Technology (1961).

7. Fabula, A. G., "The Toms Phenomenon in the Turbulent Flow of Very Dilute Polymer Solutions", (Abstract), Paper No. 119, Fourth International Congress on Rheology (1963).

8. Oldroyd, J. G., "Non-Newtonian Effects in Steady Motion of Some Idealized Elastico-Viscous Liquids", A 245: 278 (1958).

9. Rivlin, R. S: Proc. Roy. Society of London, A 193, pp. 260-281 (1948).

10. Rivlin, R. S. and Ericksen, J. Rotional Mech. and Analysis, 4:323 (1955).

11. Coleman, B. D., and Noll, Walter, "Recent Results in the Continium Theory of Viscoelastic Fluids", Annals N.Y.A.S., Vol. 89, p. 672 (1961).

12. Shertzer, C. R., and Metzner, A. B., "Measurement of Flow Properties of Viscoelastic Materials" (Abstract). Paper No. 57, Fourth International Congress on Rheology (1963).

13. Thurston, G. B., "Measurement of the Acoustic Impedance of a Viscoelastic Fluid in a Circular Tube", Jour. Acous. Soc. Amer., Vol. 33, No. 8, pp. 1091 - 1095 (1961).

14. Walters, K., "The Motion of an Elastico-Viscous Liquid Contained Between Coaxial Cylinders (II)", Quart. Journ. Mech. and Applied Math., Vol. XIII, Pt. 4, (1960).

15. Prandtl, L.: in W. F. Durand (Ed.) Aerodynamic Theory, Vol. III, Div. G, "The Mechanics of Viscous Fluids", (1934).

16. Van Driest, E. R., and Blumer, C. B., "Boundary Layer Transition: Free-stream Turbulence and Pressure Gradient Effects", A. I. A. A. Journal Vol. 1, No. 6, pp. 1303 - 1306 (1963).

17. Schlicting, H., Boundary Layer Theory, McGraw Hill Book Co., New York (1955).

18. Merill, E. W., "Viscometric Classification of Polymer Solutions", Ind. and Eng. Chem., Vol. 51, No. 7, 868 - 870 (1959).

19. Nikuradse, J., VDI "Gesetzmäsigkeiten Der Turbulenten Strömung in Glatten Rohren", Forschungsheft 356 (1932).